* Digital control - session

Analysis

c/c eqn $|ZI-A| = 0$

T.F $\quad C(ZI-A)^{-1}B$

Controllability

← أقدر أتحكم في ال $\begin{pmatrix}\text{Sys.}\\ \text{states}\end{pmatrix}$ بـ (Control i/p)

$$M_c = [B \quad AB \quad A^2B \ldots A^{n-1}B]$$

$$|M_c| \neq 0$$

→ control input

$$x(n) = M_c^{-1}\,u$$

← لازم يكون قيمة M_c موجودة عشان يطلع قيمة.

فتكون M_c^{-1} موجوده.

observability

ـــ أشوف (sys. states) من خلال مجموعة من ال(measuring)

$$M_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \qquad M_o \neq 0$$

Ex

$$x(k+1) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} x(k)$$

c/c eqn

$$|zI - A| = \begin{bmatrix} z & 0 & 0 \\ 0 & z & 0 \\ 0 & 0 & z \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} = 0$$

$$= \begin{vmatrix} \overset{+}{z} & \overset{-}{-1} & \overset{+}{0} \\ \overset{-}{0} & \overset{+}{z} & \overset{-}{-1} \\ \overset{+}{1} & \overset{-}{2} & \overset{+}{z+3} \end{vmatrix}$$

$$|zI - A| = z\begin{vmatrix} z & -1 \\ 2 & z+3 \end{vmatrix} - (-1)\begin{vmatrix} 0 & -1 \\ 1 & z+3 \end{vmatrix} = 0$$

$$z^3 + 3z^2 + 2z + 1 = 0$$

Controllability $\quad M_c = (B \quad AB \quad A^2B)$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$$

$$A^2B = A \cdot AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 7 \end{bmatrix}$$

$$M_c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -3 \\ 1 & -3 & 7 \end{bmatrix}$$

$$|M_c| = 1\begin{vmatrix} 0 & 1 \\ 1 & -3 \end{vmatrix} = -1$$

Observability $\quad M_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$

$$C = (1 \quad 0 \quad 1)$$

$$CA = (1 \quad 0 \quad 1)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} = \cancel{\begin{pmatrix} -1 \\ -1 \\ -3 \end{pmatrix}}$$

$$(-1 \quad -1 \quad -3) \qquad\qquad (-1 \quad -1 \quad -3)$$

$$CA^2 = CA \cdot A = \cancel{\begin{pmatrix} -1 \\ -1 \\ -3 \end{pmatrix}}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} = (3 \quad 5 \quad 8)$$

$$M_o = \begin{pmatrix} 1 & 0 & 1 \\ -1 & -1 & -3 \\ 3 & 5 & 8 \end{pmatrix}$$

$$|M_o| = 1\begin{vmatrix} -1 & -3 \\ 5 & 8 \end{vmatrix} + 1\begin{vmatrix} -1 & -1 \\ 3 & 5 \end{vmatrix}$$

$$= 5$$

c/c eqn

$$Z^3 + 3Z^2 + 2Z + 1 = 0$$

stability

Jury Test

Routh array

Pole location

A: system matrix $\longrightarrow A_{diagonal} = T^{-1} A T$

$$\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$\lambda \rightarrow$ eigen values

T.F $\quad \dfrac{C \, \mathrm{adj}(ZI - A) B}{|ZI - A|}$

$$|ZI-A| = \begin{bmatrix} Z & -1 & 0 \\ 0 & Z & -1 \\ 1 & 2 & Z+3 \end{bmatrix}$$

$$\operatorname{adJ}(ZI-A) = \begin{bmatrix} \overset{+}{Z^2+3Z+2} & \overset{-}{1} & \overset{+}{-Z} \\ \overset{-}{-Z-3} & \overset{+}{Z^3+3Z} & \overset{-}{2Z+1} \\ \overset{+}{1} & \overset{-}{-Z} & \overset{+}{Z^2} \end{bmatrix}^T$$

ـه كل حد بشيل صفه وعمود وأجيب المحدد اللى هيطلع

و بعدين الإشارة فى خطوة تانية.

$$= \begin{bmatrix} Z^2+3Z+2 & -1 & -Z \\ Z+3 & Z^3+3Z & -2Z-1 \\ 1 & Z & Z^2 \end{bmatrix}^T$$

$$\operatorname{adj}(zI - A) * B = \begin{bmatrix} z^2+3z+2 & z+3 & 1 \\ -1 & z^2+3z & z \\ -z & -2z-1 & z^2 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ z \\ z^2 \end{bmatrix}$$

$$C \operatorname{adj}(zI - A) B = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \begin{bmatrix} 1 \\ z \\ z^2 \end{bmatrix} = 1 + z^2$$

$$T.F = \frac{z^2+1}{z^3+3z^2+2z+1}$$

Time response

$$\phi(z) = (zI - A)^{-1}$$

$$x(z) = \phi(z) * z * x(0) + \phi(z) B\, u(z)$$

$$x(k) = z^{-1}[x(z)]$$

$$y(k) = C\,x(k) + D\,u(k)$$

Ex

$$x(k+1) = \begin{pmatrix} 0.6 & 1 \\ 0 & 0.7 \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} 1 & 1 \end{pmatrix} x(k) + 2\,u(k)$$

$$x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \; ; \; x(k) = 1$$

$$\phi(z) = (zI - A)^{-1} = \left[\begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix} - \begin{pmatrix} 0.6 & 1 \\ 0 & 0.7 \end{pmatrix}\right]^{-1}$$

$$= \begin{pmatrix} z + 0.7 & 1 \\ 0 & z - 0.6 \end{pmatrix} \frac{1}{(z-0.6)(z-0.7)}$$

$$X(z) = \frac{Z}{(Z-0.6)(Z-0.7)} \begin{bmatrix} Z-0.7 & 1 \\ 0 & Z-0.6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} +$$

$$\frac{Z}{(Z-1)(Z-0.6)(Z-0.7)} \begin{bmatrix} Z-0.7 & 1 \\ 0 & Z-0.6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{Z}{(Z-0.6)(Z-0.7)} \begin{bmatrix} 1 \\ Z-0.6 \end{bmatrix} + \frac{Z}{(Z-1)(Z-0.6)(Z-0.7)} \begin{bmatrix} 1 \\ Z-0.6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{Z}{(Z-0.6)(Z-0.7)} \\ \frac{Z}{Z-0.7} \end{bmatrix} + \begin{bmatrix} \frac{Z}{(Z-1)(Z-0.6)(Z-0.7)} \\ \frac{Z}{(Z-1)(Z-0.6)} \end{bmatrix}$$

- كان ممكن تاخد المصفوفة عامل مشترك و تجمع الكسور ثم تدخلها للمصفوفة وتعمل (Partial fraction) لكن (P.F) مش بيتعمل خارج المصفوفة.

- اما تعمل (P.F) لازم تخرج ال (Z) بره ال P.F

ex $Z \left[\frac{A_1}{Z+a} + \frac{A_2}{Z+b} \right]$

$$X(z) = \begin{bmatrix} \frac{-10z}{z-0.6} + \frac{10z}{z-0.7} \\ \frac{z}{z-0.7} \end{bmatrix} + \begin{bmatrix} \frac{8.33z}{z-1} + \frac{25z}{z-0.6} - \frac{33.33z}{z-0.7} \\ \frac{3.33z}{z-1} - \frac{3.33z}{z-0.7} \end{bmatrix}$$

$$x(K) = \begin{bmatrix} -10(0.6)^K + 10(0.7)^K \\ (0.7)^K \end{bmatrix} + \begin{bmatrix} 8.33 + 25(0.6)^K - 33.33(0.7)^K \\ 3.33 - 3.33(0.7)^K \end{bmatrix}$$

$$= \begin{pmatrix} 15(0.6)^K - 23.33(0.7)^K + 8.33 \\ 3.33 - 2.33(0.7)^K \end{pmatrix}$$

$$y(K) = (1 \quad 1)\, x(K) + 2$$

$$= 15(0.6)^K + 23.33(0.7)^K + 8.33 + 3.33 - 2.33(0.7)^K + 2$$

$$y(K) = 13.66 + 15(0.6)^K - 25.66(0.7)^K$$